Time Speed and Distance - III (Challenging Questions)

1. Amit is chasing Akash in his car on Delhi-Agra expressway. He observes that when he is just 100m away from a dark tunnel, Akash has already entered the tunnel and covered some distance inside. When Amit just emerges from the tunnel, he can see Akash 80 m ahead of him. Amit is 12.5% faster than Akash.

1. What fraction of the length of the tunnel Akash must have covered, When Amit was 100m away from the tunnel?

2. If Akash has covered only 80m inside the tunnel when Amit is 100m away from the tunnel, then Amit catches Akash exactly 16s after Amit has emerged from the tunnel. Find the time required by Amit to cross the tunnel.

Sol 1: Let the length of the tunnel = x and Akash has covered kth part of the tunnel. So Akash is 'kx' distance inside the tunnel when Amit is 100m away from the tunnel. Here k is a proper fraction i.e., Given that Amit is 12.5% or 112.5/100 times faster than Akash. So their speeds ratio = 9/8

From the given condition

$\Rightarrow \dfrac{(100+x)}{(x-kx)+80} = \dfrac{9}{8}$

Or, 800 + 8x = 9x - 9kx + 720

Or, x - 9kx = - 80

Or, x (1 - 9k) = - 80

As x cannot be negitive 1-9k should be negitive

so 1 - 9k < 0

So k $< \dfrac{1}{9}$

Sol 2: It was given Akash has covered 80 m

$\Rightarrow \dfrac{(100+x)}{(x-80)+80} = \dfrac{9}{8}$

$\Rightarrow$ 800 + 8x = 9x

$\Rightarrow$ x = 800

Given that Amit takes 16 s to catch Akash after he emerged from the tunnel. So to gain 80 m, Amit has taken 16 s, so he gains 80/16 = 5m every second.

i.e., Speed of Amit - Speed of Akash = 5 m, But we already know that Amit / Akash = 9/8,

Solving these two we get Amit speed = 45 m/s

So Amit will take 800/45 sec to cross the tunnel or 160 / 9 seconds.

2. A bird starts flying from a place O towards B via A. There is no wind resistance from O to A. But there is wind resistance (in the form of uniform wind velocity) between A and B. To travel from O to B the bird takes 12 minutes, whereas to travel from B to O (via A) the bird takes 14 minutes. Had there been wind resistance between O and A also (Which equals, that between A and B), the bird would have taken only 11 minutes to travel from O to B. O, A and B are in a straight line.

Q 1: If there was the same wind resistance between O and A as between A and B, what would be the time taken by the bird to travel from B to O?

a. 15 minutes b. 15.4 minutes c. 16.2 minutes d. None of these

Q2: What is the ratio of the distance OA to the distance AB?

a. 5:4 b. 7 : 6 c. 6:5 d. None of these

Let OA = 1 km and AB = x km

Let V be the velocity (in km/min) of the bird when there is no wind resistance. Let U be the wind velocity from A to B (both from O to B)

(since timne is less from O to B than B to O)

Given that

$$\frac{1}{V} + \frac{x}{V+U} = 12 \quad \text{.............(1)}$$

$$\frac{1}{V} + \frac{x}{V-U} = 14 \quad \text{..............(2)}$$

$$\frac{1}{V+U} + \frac{x}{V+U} = 11 \quad \text{...(3)}$$

Eq (1) - Eq (3) = $\frac{1}{V} - \frac{x}{V+U} = 1 \Rightarrow \frac{U}{V(V+U)} = 1$ ...... (4)

Eq(2) - Eq (1) = $\frac{x}{V-U} - \frac{x}{V+U} = 2$

$$\frac{x(2U)}{(V-U)(V+U)} = 2 \Rightarrow \frac{x(U)}{(V-U)(V+U)} = 1 \quad \text{........(5)}$$

Equation (4) and (5)

$$\frac{1}{V} = \frac{x}{V-U}$$

Substituting this in equation (2) we get

$$\frac{1}{V} = 7, \frac{x}{V-U} = 7$$

So, V = 1/7, x = 7(V-U)

From equation (4), U = V(V+U)

U = 1/49 + 1/7U

U = 1/42

x = 7(1/7 - 1/42) = 5/6

we get x = 5/6

U = 1/42

V = 1/7

Time taken by the bird to travel from B to O with wind resistance throughout =

$$\frac{1+x}{V-U} = \frac{1+\frac{5}{6}}{\frac{1}{7}-\frac{1}{42}} = \frac{11/6}{5/42} = 15.4 \text{ min}$$

Alternate Method:

Assume Speed of the wind is 1 unit and bird speed is n' units

Given that $\frac{OA}{n} + \frac{OB}{n+1} = 12$ ....(1)

$\frac{OA}{n} + \frac{OB}{n-1} = 14$ ....(2)

$\frac{OA}{n+1} + \frac{OB}{n+1} = 11$ ....(3)

From equation (3) we get (OA + OB) = (n+1) 11

OA + OB is a multiple of 11.

Now look at the second quesiton, The distances ratios were given. From the given options, 6:5 satisfies above condition. (may be n = 1/11 may also satisfies the condition, But we have to take chances!)

Assume OA, OB are 6k, 5k and substituting in equation (3)

11k = (n+1)11 ⇒ n = k-1

Substituting this value in equation (1) we get

$$\frac{6k}{k-1} + \frac{5k}{k} = 12 \Rightarrow \frac{6k}{k-1} = 7 \Rightarrow k = 7$$

So n = 6.

If there is wind resistance from O to A also, the bird will take $\frac{OA}{n-1} + \frac{OB}{n-1}$ min

Substituting above values, $\frac{6(7)}{6-1} + \frac{5(7)}{6-1} = \frac{77}{5} = 15.4$ min

3. A and B run around a circular track running in the same direction. They started at the same point and at the same time. A's speed is four times B's speed. Both of them have different number of coins with them. Whenever one of them overtakes the other, the former gives the latter as many coins as the latter already had, so that the number of coins with the latter doubles. However, If a person overtakes the other at the starting point, then the latter gives half the number of coins with him to the former. It is known that B started with one coin, and A finished with 70 coins after a certain number of rounds.

Q 1: Which of the following could be the number of coins A started with?

a. 101 b. 122 c. 130 d. 180

Q2: After the transfer of tokens at the end of $3K^{th}$ meeting (where K is a positive integer), B has 16 tokens with him. How many tokens would B have had after the transfer of tokens at the end of $(3K-2)^{th}$ meeting point?

a. 8 b. 16 c. 3K + 2 d. 3K+5

Solution:

It was given that speed's of A and B are in the ratio: 4 : 1. Recap hat, when two persons are running in the same direction, one can overtake the other, when one can covered one round more than the other.

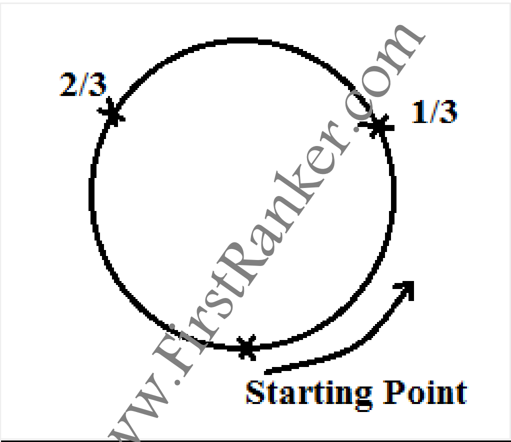
i.e., In the time B covers 1 rounds, A covers 4 round.

In the time B covers 1 round, A covers 3 rounds Extra.

In the time B covers 1/3 round, A covers 1 round Extra

So A meets B, each time B covers 1/3rd of the round. So A can overtake B at three different points on the circular track and one of the meeting point is starting point.

So A gives coins two times before he takes from B at starting point.



Now let us genaralize the situation, by assuming that A starts with 'a' coins, and B starts with 'b' coins.

Meeting	A (starts with a coins)	B (starts with b coins)	Transactions
1	a - b	2b	A gives 'b' coins to B
2	a - 3b	4b	A gives '2b' coins to B
3	a - b	2b	B gives '2b' coins to A
4	a - 3b	4b	A gives '2b' coins to B
5	a - 7b	8b	A gives '4b' coins to B
6	a - 3b	4b	B gives '4b' coins to A
7	a - 7b	8b	A gives '4b' coins to B
8	a - 15b	16b	A gives '8b' coins to B
9	a - 7b	8b	B gives '8b' coins to A

From the table, the coins with B are in the form of 2^n b and the number of coins with A in the form of **a - (2^n - 1)b**

Given that A ended up with 70 coins.

So a - (2^n - 1)b = 70

Substituting b = 1

a - (2^n - 1) =70

a = 70 + (2^n - 1)

The number of coins A has will be of the form 70 + 2^n - 1 and only option (1) satisfies this condition for n = 5

Sol 2: From the above table, we can conclude that the number of coins with B in the first and third meeting, fourth and sixth meeting ...are same. So the number of coins with B in 3K, 3K - 2 rounds are same. So correct option is (2)